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*Regulus*, 16 *Cygni*, *Castor*, $\nu^1$, $\nu^2$ *Draconis*, 1 *Pegasi*, $\iota$ *Boötis*, 93 *Leonis*, $\mu$ *Boötis*, $\theta^1$, $\theta^2$ *Tauri*, $\epsilon$ and 5 *Lyræ*, $\alpha$ *Tauri*, 6 and 8 *Vulpec.*, 16 and 17 *Draconis*, 33 and 34 *Ophiuchi*, $\alpha^1$ and $\alpha^2$ *Capricorni*; also P. III. 241 and 50 *Persei* (see O Σ and Argelander).

In addition to the above, a good many pairs, whose changes in angle and distance can be explained by the proper motion of the principal star, were long ago examined by Struve (*Pos. Med.*) Perhaps these should go into the list. Also the "higher systems" of Mädler would probably not be misplaced in it (see *Untersuchungen über die Fixstern-Systeme*.

*Mr. E. Crossley's Observatory,*
*Bermerside, Halifax,*
*December* 1876.

## *On an Oversight in the Mécanique Céleste, and on the Internal Densities of the Planets.* By George H. Darwin, M.A., Fellow of Trinity College, Cambridge.

(*Communicated by J. W. L. Glaisher, F.R.S.*)

In the following paper an endeavour is made to point out an inconsistency, which appears to have escaped the notice of Laplace, in his determination of the precessional constants of the planets *Jupiter* and *Saturn*. From this I have been led on to speculate on the law of internal density of those planets, and of *Mars*, and to make some reference to the ellipticities of *Mercury* and *Venus*.

### 1. *Laplace's Law of the Internal Density of the Planets.*

In the investigation of the figure of the Earth, Laplace assumed that, in molten rock, the hydrostatic pressure *plus* a constant varies as the square of the density. The result of this assumption is that, after the consolidation of a planet, the density of any stratum of mean radius $r$ is given by the law $\frac{F}{r}\sin\frac{r\theta}{a}$, where $a$ is the mean radius of the surface, and $\theta$ and F are constants.

Throughout the rest of this paper, besides the foregoing, the following notation is used:—

$\alpha$, $\beta$, the equatoreal and polar radii;
$\epsilon$ the ellipticity of the surface;
$m$ the ratio of the centrifugal force of the planet's rotation at the distance $a$ to the mean pure gravity;

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$f$ the ratio of the mean to the surface density;

C, A, the greatest and least principal moments of inertia of the planet;

P or $\frac{C-A}{C}$ the precessional constant; and

M the mass.

Other symbols will be defined as they arise.

Then the results of the Laplacian law may be embodied in the following equations*:—

$$\frac{f}{3} = \frac{1}{\theta^2} - \frac{\cot\theta}{\theta} \quad \dots\dots\dots\dots \quad (1)$$

$$\frac{5m}{2\epsilon} = \frac{f\theta^2}{3(f-1)} - \frac{3}{f} \quad \dots\dots\dots\dots \quad (2)$$

$$C - A = \frac{2}{3}\left(\epsilon - \frac{m}{2}\right) Ma^2 \quad \dots\dots\dots\dots \quad (3)$$

$$C = \frac{2}{3}\left\{1 - \frac{6(f-1)}{f\theta^2}\right\} Ma^2 \quad \dots\dots\dots \quad (4)$$

$$\frac{\epsilon - \frac{m}{2}}{P} = 1 - \frac{6(f-1)}{f\theta^2} \quad \dots\dots\dots\dots \quad (5)$$

Now, if (following Thomson and Tait) we take for the Earth $f = 2{\cdot}1$, which corresponds with $\theta = 144°$, we have

$$P = 1{\cdot}994\left(\epsilon - \frac{m}{2}\right) \quad \dots\dots\dots\dots \quad (6)$$

$$\frac{5m}{2\epsilon} = 2{\cdot}562 \quad \dots\dots\dots\dots \quad (7)$$

Hence, in any planet where the law of density is the same as in the Earth (i.e., $f = 2{\cdot}1$, $\theta = 144°$), we should find (7) satisfied, and (6) would give the precessional constant.

In order to illustrate the Laplacian law, the following table has been constructed, giving the values of $f$ and $\frac{5m}{2\epsilon}$ for every 10° of $\theta$, with the omission of a few at the early part, which are rather troublesome to calculate, and are of little value.

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* See Thomson and Tait, *Nat. Phil.* §§ 824, 827. Equation (3) is independent of Laplace's assumption.

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| $\theta$ | $f$ | $\frac{5m}{2\varepsilon}$ |
|---|---|---|
| 0° | 1·0000 | 2·000 |
| 10 | | |
| 20 | 1·0082 | |
| 30 | 1·0188 | |
| 40 | 1·0341 | 2·029 |
| 50 | 1·0548 | 2·046 |
| 60 | 1·0817 | 2·067 |
| 70 | 1·1161 | 2·094 |
| 80 | 1·1600 | 2·126 |
| 90 | 1·2159 | 2·165 |
| 90° | 1·2159 | 2·165 |
| 100 | 1·2879 | 2·213 |
| 110 | 1·3827 | 2·270 |
| 120 | 1.5109 | 2·338 |
| 130 | 1·6922 | 2·422 |
| 140 | 1·9657 | 2·525 |
| 150 | 2·4225 | 2·652 |
| 160 | 3·3363 | 2·813 |
| 170 | 6·0750 | 3·019 |
| 180 | infin. | 3·290 |

This table shows how $\frac{5m}{2\varepsilon}$ increases, as the planet passes from homogeneity to infinitely small surface density; $\frac{5m}{2\varepsilon}$ can never be less than 2, nor greater than 3·290. Although it is not strictly involved in the subject of this paper, I may point out that the first column of the table may be employed to calculate the ellipticity of any internal stratum. I have shown elsewhere* that the ellipticity $e$ of any stratum of mean radius $\frac{\theta'}{\theta}a$ is given by,—

$$\frac{e}{\varepsilon} = \left(\frac{\theta}{\theta'}\right)^2 \frac{f\,(f'-1)}{f'\,(f-1)},$$

where $f$, $\theta$ refer to the surface, and $f'$ is the value corresponding to $\theta'$ in the table. For example, to find the ellipticity of the Earth's stratum of mean radius $\frac{5}{12}a$, we must take $\theta' = 60°$, because $\theta = 144°$; then from the table $f' = 1·0817$, and

$$e = \varepsilon\left(\frac{12}{5}\right)^2 \cdot \frac{2·1 \times ·0817}{1·0817 \times 1·1} = \frac{\varepsilon}{1·204}$$

$$= \frac{1}{357}, \text{ when } \varepsilon = \frac{1}{297}.$$

## 2. *Jupiter's Precessional Constant.*

To determine the precessional constant Laplace uses the following argument:—"If we suppose the densities of the strata of Jupiter and the Earth, at distances proportional to the diameters of these planets, to be in a constant ratio to each other. . . In this hypothesis, if both planets be fluid, their ellipticities will be,

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* *Messenger of Mathem.*, Nov. 1876, p. 109. The notation is, however, different.

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as in [2068 *k*, etc., III. v. § 43] proportional to the respective values of $\phi$,* corresponding to each of them; or to their ellipticities, if they be homogeneous. If we suppose the ratio to obtain in their actual state, and we have seen in [2069, III. v. § 43] that this is nearly conformable to observation, *then the values of* $\frac{2\,C - A - B}{C}$ *will be, for each of these planets, respectively proportional to the ellipticities corresponding to the case of homogeneity.* These ellipticities by the same article [2068″], are as ·10967000 to 0·00433441." † He uses ·00291193 as the Earth's precessional constant, and thence deduces $\frac{2\,C - A - B}{C} = {}$·14735, or in my notation P = ·07368.

By the theory of the perturbations of the Satellites, he finds $\epsilon - \frac{m}{2} = {}$·0219013; ‡ this value does involve any assumption as to the law of internal density, except, I conceive, that the strata of equal density must be nearly spherical.§

Then by the periodic time of the Fourth Satellite, and its observed distance from *Jupiter*, he finds $m = {}$·0987990; whence $\epsilon = {}$·0713008.‡

Using these values of $m$ and $\epsilon$, we get $\frac{5m}{2\epsilon} = 3{\cdot}4642$. Now Laplace does not seem to have noticed that these values are not only incompatible with the identity of the law of internal density in the *Earth* and *Jupiter*, but are also incompatible with that form of law at all; for, as appears from the table, $\frac{5m}{2\epsilon}$ can never be greater than 3·290.

My attention was first drawn to the point by observing that, if there were this identity of law of density, the precessional constant of *Jupiter* ought to be nearly twice $\epsilon - \frac{m}{2}$, or ·0438; whereas Laplace, by a different method, founded on the same assumption, finds it to be ·0737. This of course indicated that the assumption was untenable.

In view of this discrepancy, it will be well to go over Laplace's work again by the light of later, and presumably better observations.

The distance of the Fourth Satellite is open to some doubt, but the following has seemed to me the best value attainable. Bessel gives the apparent distance at the planet's mean distance the angles as 498·8663″.**

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* The $m$ of this paper.

† Bowditch, Trans. of *Méc. Cél.* VIII. vii., § 23.

‡ VIII. ix. § 27.

§ See Pratt's *Fig. of Earth,* Arts. 90–94.

** *Astron. Untersuch. Bestim. der Masse des Jupiter,* Königsberg 1842.

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M. Kaiser gives, as the result of a long series of his own observations when united with those of Bessel, that the polar and equatoreal diameters of *Jupiter* subtend at the same distance $35''\!\cdot\!170$ and $37''\!\cdot\!563$ respectively.*

Hence the mean distance D of the Satellite is 26·5616 equatoreal radii; or $D = 26\cdot5616\, a$; T, the satellite's periodic time is 16·68902 m. s. days, $t$, *Jupiter's* sidereal day is ·4135 m. s. days. Therefore

$$m = \left(\frac{T}{t}\right)^2 \left(\frac{a}{D}\right)^3 (1-\epsilon)$$

$$= \cdot0869258\,(1-\epsilon) = m'\,(1-\epsilon) \text{ suppose.}$$

Professor J. C. Adams informs me that M. Damoiseau has recalculated the values of $\epsilon - \frac{m}{2}$, and of the masses of the Satellites, basing his work on better observations than those which were available to Laplace.† He has taken, however, certain coefficients from Laplace, which ought to have been recalculated. This work has been completed by Professor Adams himself, and he has obtained the following value for $\epsilon - \frac{m}{2}$, of which he has, with very great kindness, allowed me to make use, viz.,

$$\epsilon - \frac{m}{2} = \cdot0216623‡$$

Substituting then for $m$, we have, for the ellipticity of *Jupiter*,—

$$\epsilon = \frac{\frac{m'}{2} + \cdot0216623}{1 + \frac{m'}{2}} = \cdot06241 = \frac{1}{16\cdot022},$$

and

$$m = m'\,(1-\epsilon) = \cdot081501.§$$

Laplace remarks** that the ellipticity of *Jupiter* may be found

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* *Astron. Nachr.* 48, p. 111.

† *Tables des Satellites de Jupiter.* Bur. des Long., Bachelier, Paris 1836.

‡ The corresponding masses of the Satellites multiplied by 1000 are found by him to be:

·283113
·232355
·812453
·214880

They seem to bear hardly any relation to the ordinarily received values.

§ This value may be compared with ·08163 found by Plana, *Ast. Nach.* 36, p. 155.

** VIII. ix. § 27.

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by this method with greater accuracy than by the best observations. It is interesting, therefore, to observe how closely this value agrees with that of all the later observations. The following are the values given for the reciprocal of the ellipticity by—Secchi, 16·06 [*Ast. Nach.* 43, p. 142]; Kaiser, (i) 15·36, (ii) 15·98 [*Ast. Nach.* 45, p. 211]; Bessel and Kaiser, 15·70; Bessel, 15·73; Secchi, 15·99 [*Ast. Nach.* 48, p. 111]; Schmidt, 15·6 [*Ast. Nach.* 65, p. 102]; Main, 16·84 [*Month. Not. Ast. Soc.* 16, p. 142]. The reductions were in several cases made by myself.

It is, nevertheless, a curious coincidence that the value of $\varepsilon$ found by Laplace agreed well with the older observations of Struve, who found $\varepsilon = \frac{1}{13\cdot74}$. [*Ast. Nach.* 45, p. 211.]

Using the above value of $m$, I find (following Plana*) that on the hypothesis of homogeneity $\frac{a}{\beta} = 1\cdot1164$ (compare with $1\cdot116515$ of Plana), and therefore the homogeneous ellipticity is $\cdot10405$. But the homogeneous ellipticity of the Earth is $\frac{1}{230\cdot433}$ (Plana); and the Earth's precessional constant is $\cdot003272$; hence, following Laplace's method, we should have for *Jupiter*,

$$\begin{aligned} P &= \cdot10405 \times 230\cdot433 \times \cdot003272 \\ &= \cdot078451. \end{aligned}$$

But if Laplace's assumption as to the internal density of *Jupiter* were justifiable, we ought to have by equation (6),

$$\begin{aligned} P &= 1\cdot994\left(\varepsilon - \frac{m}{2}\right) = 1\cdot994 \times \cdot02166 \\ &= \cdot043. \end{aligned}$$

Thus these new values of the quantities involved leave as wide a discrepancy as before between the two values of P. Therefore, Laplace's method cannot be justified.

### 3. *The Internal Density of Jupiter.*

But this unjustifiability may be seen in another way, for the new values of $m$ and $\varepsilon$ give $\frac{5m}{2\varepsilon} = 3\cdot2646$; and reference to equation (7) shows that this differs widely from the corresponding value for the Earth. It is not, however, inconsistent with the Laplacian form of law, as was the same function when Laplace's values of $m$ and $\varepsilon$ were used. In fact, I find, after some rather troublesome arithmetic, that $\frac{5m}{2\varepsilon} = 3\cdot2646$ corresponds to $\theta = 179^\circ\ 11'\ 20''$ nearly, and to $f = 68$.

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* All future references to M. Plana are to his paper in *Astr. Nachr.* 36, p. 154.

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Using these values of $\theta$ and $f$, I find by equation (5) that *Jupiter's* precessional constant $= 2{\cdot}528\left(\varepsilon - \frac{m}{2}\right) = {\cdot}0548$; and this I take to be far nearer the truth than the value assigned by Laplace.

On account of the uncertainty in the determination of the quantity $m$, the above values of $\theta$ and $f$ can, of course, have no claim to precision. In fact, if following Herschel, Loomis, and others, we take $\frac{D}{a} = 26{\cdot}99835$, we should find $\varepsilon = \frac{1}{16{\cdot}52}$, $m = {\cdot}0778$, and $\frac{5m}{2\varepsilon} = 3{\cdot}211$. This last corresponds to $\theta$ about 177° 42′, and $f$ about 24.

The value of the precessional constant will, however, be but little changed, being $2{\cdot}489\left(\varepsilon - \frac{m}{2}\right) = {\cdot}0539$.

Now it seems reasonable *à priori* to assume that the law of internal density within *Jupiter* is of the same *nature* as in the Earth, and from that assumption it follows that the surface density of *Jupiter* is vastly less than the mean: for it must, I think, be admitted that numerical values can be assigned to $m$ and $\varepsilon$ with some precision.

But the true meaning of this result would seem to be that the Laplacian law of density is not exact for *Jupiter*, but that that planet must be very much denser in the centre than at the surface. Is it not possible that *Jupiter* may still be in a semi-nebulous condition, and may consist of a dense central part with no well-defined bounding surface? Does not this view accord with the remarkable cloudy appearance of the disk, and the remarkable belts?

### 4. *Saturn.*

The preceding method cannot be applied very satisfactorily to the case of *Saturn*, on account of the uncertainty in all the quantities which enter into the determination of $m$ and $\varepsilon$; but the balance of evidence appears to me decidedly in favour of his surface density being far less than the mean.

$m$ may be best determined from the motion of *Japetus*.

There seems almost complete unanimity as to his periodic time, and I take T = 79·3296 m. s. days.

Two values are assigned for the sidereal day of *Saturn*, viz., ·4370 (Hansen), and ·4278 (Herschel) m. s. days, the latter being more generally accepted; I take then $t = {\cdot}430$.

There is a wide difference of opinion as to the mean distance D of *Japetus*, the estimates varying from $57{\cdot}4\,a$ (Jacob, *Mem. Astr. Soc.* vol. xxviii.) to $64{\cdot}359a$ (used by Herschel, Loomis, and others, and most generally accepted). I take $D = 62\,a$.

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The ellipticity seems fairly well determined at $\frac{1}{11}$.*

Using these quantities in the formula

$$m = \left(\frac{T}{t}\right)^2 \left(\frac{a}{D}\right)^3 (1-\epsilon),$$

I find

$$m = \cdot 131270 - \cdot 61056\,\delta t - \cdot 00635\,\delta\left(\frac{D}{a}\right) - \cdot 14281\,\delta\epsilon.$$

(i) If all the small variations are zero, and $\epsilon = \frac{1}{11}$, we get $\frac{5m}{2\epsilon} = 3\cdot 610$.

(ii) Loomis, *Pract. Astron.*, Tables 33, 34.

$$t = \cdot 437, \quad \frac{D}{a} = 64\cdot 359, \quad \epsilon = \frac{1}{10},$$

$$\delta t = \cdot 007, \quad \delta\frac{D}{a} = 2\cdot 359, \quad \delta\epsilon = \cdot 01909,$$

then, $m = \cdot 1093, \quad \frac{5m}{2\epsilon} = 2\cdot 732.$

(iii) Hind's *Solar System*, pp. 103–4.

$$t = \cdot 4278, \quad \frac{D}{a} = 60\cdot 3436, \quad \epsilon = \frac{1}{10\cdot 28}$$ according to Bessel,

$$\delta t = -\cdot 0022, \quad \delta\frac{D}{a} = -1\cdot 6564, \quad \delta\epsilon = \cdot 00637,$$

then, $m = \cdot 1422, \quad \frac{5m}{2\epsilon} = 3\cdot 66.$

If we take $\epsilon = \frac{1}{9\cdot 23}$ from the *Greenwich Observations*, $\frac{5m}{2\epsilon}$ will be slightly smaller.

(iv) Captain Jacob's *Madras Observations* give $\frac{D}{a} = 57\cdot 4$, and $\delta\frac{D}{a} = -4\cdot 6$, this with $\epsilon = \frac{1}{11}$ and all the other small variations zero, gives $m = \cdot 160$ and $\frac{5m}{2\epsilon}$ very nearly $4\frac{1}{4}$.

No doubt other values might be assigned to $\frac{5m}{2\epsilon}$ with equal

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[*] I find the following in a paper by Mr. Grant (*Month. Not. Ast. Soc.* xiii. p. 195), where he reduces observations of apparent ellipticity to their real values, viz., Lassell $\frac{1}{11\cdot 05}$, Main $\frac{1}{9\cdot 227}$, De La Rue $\frac{1}{10\cdot 611}$. The mean of all Bessel's observations would seem to be $\frac{1}{10\cdot 20}$, but Mr. Hind (see below) assigns a slightly different value as Bessel's result.

Sir W. Herschel found $\frac{1}{10\cdot 368}$ (see p. 79 of above vol.), but $\frac{1}{10\cdot 384}$ is also attributed to him on the same page.

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plausibility, but I think (ii) and (iv) contain extreme values in the two directions, and the truth probably lies at some intermediate point. It is noticeable that even the value (ii) indicates a larger ratio of mean to surface density than in the Earth, namely 2·809 (corresponding to $\theta = 155^\circ\ 14'$), whilst all the other values are far outside the limits of admissibility under the Laplacian law.

I have tried also to find $m$ from the motion of *Titan*. Taking

$$\frac{\mathrm{D}}{a} = 21, \quad \mathrm{T} = 15\cdot 9454, \quad t = \cdot 43, \quad \varepsilon = \frac{1}{11},$$

I find

$$m = \cdot 13648 - \cdot 1174\,\delta t - \cdot 0195\,\delta\frac{\mathrm{D}}{a} - \cdot 1485\,\delta\varepsilon.$$

If the small increments are zero, $\frac{5m}{2\varepsilon} = 3\cdot 75$; and if the various possible values be assigned to them, the results will be found to be very similar to those given above.

The value of $m$ assigned by Laplace is ·165970,[*] and this, together with the ellipticities $\frac{1}{9}$ and $\frac{1}{12}$, gives $\frac{5m}{2\varepsilon}$ equal to 3·734 and 4·979 respectively, either value quite incompatible with the Laplacian law of density. The ellipticity of the planet had apparently not been observed in his time, and the numbers used by him involve an ellipticity of $\frac{1}{68\cdot 6}$, which we now know is far too small.

Laplace makes *Saturn's* precessional constant $\frac{1}{2}$ of ·27934 or ·13967 (VIII. xvii. § 37), but this value is inconsistent with any of those given above for $m$ and $e$.

Taking $\varepsilon = \frac{1}{11}$, $m = \cdot 1313$, we have $\varepsilon - \frac{m}{2} = \cdot 0252$. It can hardly be supposed that *Saturn* is more nearly homogeneous than is the Earth, and if the law of density were the same we should, by (6), have $\mathrm{P} = \cdot 050$.

On the other hand, if the Laplacian law of density were to hold good, and if the surface density were zero—of course an ideal case[†]—we should have by (5),

$$\frac{\varepsilon - \frac{m}{2}}{\mathrm{P}} = 1 - \frac{6}{\pi^2} = \frac{1}{2\cdot 5505},$$

or

$$\mathrm{P} = 2\cdot 5505 \times \cdot 0252 = \cdot 064.$$

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[*] *Méc. Cél.* VIII. xvii. § 36.

[†] It may be observed, that to be thoroughly consistent we should have in this case, $\mathrm{P} = \cdot 663m = \cdot 873\varepsilon$, because $\frac{5m}{2\varepsilon} = 3\cdot 290$; but the method is here only used to find the limiting value of the coefficient of $\varepsilon - \frac{m}{2}$.

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I conceive then, that *Saturn's* precessional constant must be very nearly equal to ·06, since the former of these values is certainly too small, whilst the latter is a little too large.

## 5. *Mars.*

*Mars* rotates in $24^h\ 37^m\ 22^s{\cdot}6$, or 1·025956 m. s. days, according to the elaborate observations of M. Kaiser.* His density is ·948 of that of the Earth.† The sidereal day is ·997270 m. s. days, and for the Earth $m$ is $\frac{1}{289{\cdot}66}$; hence for *Mars*,

$$m = \frac{1}{{\cdot}948}\left(\frac{{\cdot}997270}{1{\cdot}025956}\right)^2 \frac{1}{289{\cdot}66}$$

$$= {\cdot}0034409 = \frac{1}{290{\cdot}6}.\ddagger$$

With respect to the ellipticity of the planet the most various estimates are given. Thus, for the reciprocal of the ellipticity I find, according to Loomis (*Pract. Astr.*, Table 33) 50; Main (*Month. Not. Ast. Soc.* xvi., p. 142) 62; Kaiser (Guillemin, *Le Ciel*, p. 257) 118; Arago, doubtfully, 32; Bessel failed to detect ellipticity in observations at Königsberg; and lastly, from observations of Dr. Winnecke, to which I shall recur below, 225 (*Ast. Nach.* 48, p. 97). According to Dr. Winnecke, M. Kaiser doubts whether Mr. Main's result represents any corresponding reality.

With an ellipticity of $\frac{1}{225}$, we have $\frac{5m}{2\epsilon} = 1{\cdot}93$, and with the other values of ellipticity, a very much smaller value.

Now a reference to the table in section 1 will show that not only is even the largest of these quantities incompatible with the Laplacian law of density, but they are also incompatible with the homogeneity of the planet. We should require the planet to decrease in density towards the centre, or actually to be hollow; and on any theory of original fluidity such a state of things is almost inconceivable.

The wide discrepancy between various observations shows that the results are to be very little depended on; and for the following reasons, I venture to think that all the above values of the ellipticity are almost worthless, but that Dr. Winnecke's is by

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* See a paper by M. Julius Schmidt, *Ast. Nach.* 82, p. 333.

† Guillemin, *Le Ciel*, p. 257. The other details with respect to *Mars* are given on the authority of M. Kaiser, and this I presume is so also.

‡ Plana gives $\frac{1}{288{\cdot}84}$, using slightly different data; with Herschel's value of the density ·72, $m = \frac{1}{220{\cdot}7}$.

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far the nearest to the truth. He gives[*] as the equatoreal and polar apparent diameters, 9″·227 and 9″·186, subject to mean errors of ·045″ and ·032″ respectively.

From these measurements I find that the *probable error* of the ratio of the axes is ·0050441. The ratio itself is in decimals ·9955564, and therefore the ratio of the axes is ·9955564 ±·0050441. That is to say, it is an even chance that the ratio of the axes lies between the 1·0006 and ·990512. In other words, it results from Dr. Winnecke's observations that it is an even chance that the figure of *Mars* lies between a prolate spheroid, with an ellipticity of $\frac{1}{1667}$, and an oblate one with an ellipticity of $\frac{1}{105}$; the observed values give an ellipticity of $\frac{1}{225}$.

Dr. Winnecke himself says, "Es sprechen diese Messungen entschieden gegen eine Abplattung des *Mars*, die für das Berliner Instrument messbar wäre."

Now the telescope shows that *Mars* has an external physical constitution exceedingly like that of the Earth, and therefore there is a strong probability that its internal structure is somewhat the same. Seeing then that the observations of the best observers permit such very wide limits of error, and that the value of *m* can be assigned with some precision, I submit that there is a far better chance of being near the truth in trusting to indirect evidence than to direct observation, for the determination of the ellipticity. Assuming then that the law of internal density is the same as in the Earth, I find an ellipticity of $\frac{1}{298}$, and this, I venture to think, is nearer the truth than any of the above-quoted values derived from observation.

### 6. *Mercury, Venus.*

M. Plana assigns as the values of *m* for these planets, $\frac{1}{325 \cdot 82}$ and $\frac{1}{251 \cdot 54}$ respectively; and following the arguments used in the case of *Mars* (although they have not here equal force), we should assign to *Mercury* an ellipticity of about $\frac{1}{330}$, and to *Venus* of about $\frac{1}{260}$.

### 7. *Conclusion.*

The results arrived at may be summed up as follow:—

The values assigned by Laplace to the precessional constants of *Jupiter* and *Saturn* cannot be maintained; and the objections

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[*] *Ast. Nach.* 48, p. 97. The observations are said to be reduced for "Phase und Refraction," but I do not see it expressly stated that they are corrected for the Earth's Martian declination.

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to them remain equally strong when later observations are consulted.

Professor Adams's calculation of $\epsilon - \frac{m}{2}$ gives, as the ellipticity of *Jupiter*, $\frac{1}{16{\cdot}02}$.

The surface density of *Jupiter* is far less than the mean, and he may perhaps still be in a semi-nebulous condition.

There is a considerable probability that the like is true of *Saturn.*

The value of *Jupiter's* precessional constant is about ·0548; and that of *Saturn* probably about ·06. No dependence is to be placed on the observations of the ellipticity of *Mars*, because of their wide limits of error. Indirect evidence of the ellipticity seems safer, and we are thereby led to assign to him an ellipticity of $\frac{1}{298}$. In the cases of *Mercury* and *Venus*, we have only indirect evidence to rely on, and these ellipticities are probably about $\frac{1}{330}$ and $\frac{1}{260}$.

In conclusion, I must add that if in any case I have underrated or neglected the work of important observers, or have overrated the worth of other observations, my excuse must be my previous slight acquaintance with observational astronomy.

Postscript.—The method of this paper may perhaps give some idea of the amount of difference which might be expected to be found between the apparent equatoreal and polar diameters of the Sun.

To find $m$, the Earth may be treated as a solar satellite, and the same formula as before applied. $\frac{d}{D}$ is here the Sun's mean apparent radius (which I take as $961''{\cdot}82$) in circular measure.

$T = 365{\cdot}2569$ m. s. days.

$t$, according to Carrington, is 25·38 m. s. days.

But Mr. Christie (to whom I owe my thanks for his help in this matter) informs me that the true period may perhaps be 26 or 27 days. The following results are therefore given in duplicate; in those marked with an accent $t$ is taken as 27.

Then $m = {\cdot}00002100$, or $m' = {\cdot}00001829$.

If the law of internal density were the same as in the Earth, we should have,

$\epsilon = {\cdot}00002048$, or $\epsilon' = {\cdot}00001784$.

These correspond to differences of apparent diameter of $\cdot0394''$ and $\cdot0343''$ respectively. If the Laplacian law of density were to hold good, and if the surface density were zero, we should have, $\epsilon = {\cdot}00001596$, or $\epsilon' = {\cdot}00001390$, which correspond to differences of $\cdot0307''$ and $\cdot0267''$ respectively.

Mr. Christie informs me that the probable error in the de-

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termination of the Sun's apparent diameter is not less than 0·1″, and it appears that the observed difference of diameters is also 0·1″.

May we not conclude that the difference of the apparent diameters is probably less than ·04″, but perhaps greater than ·03″?

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## *Note on the Relative Space-penetrating Power of the Pulkowa 15-inch Refractor and Mr. Lassell's 4-foot Reflector.* By M. Otto Struve.

In his paper "On the Relative Power of Achromatic and Reflecting Telescopes" (*Monthly Notices*, vol. xxxvi. pp. 305–309) Dr. Robinson makes the following statement:—

"2. Otto Struve *thought* that the Pulkowa Achromatic of 15-inches was equal to Lassell's 24-inch Newtonian (not his 4-feet, as supposed by M. Radau): nothing definite can be built on this."

I feel sure, if Dr. Robinson had read my paper, "Ueber das von Herrn W. Lassell in Malta aufgestellte Spiegelteleskop," published in the beginning of 1864 in the *Bulletin de l'Académie de St.-Pétersbourg*, he would have considerably modified his words, and perhaps also his views. In fact, M. Radau has been quite correct in his statement. I wrote the above-mentioned paper immediately after having visited my honoured friend Mr. Lassell at Malta. Together with some other results derived from practical experience, I have given there, *in extenso*, the comparison of the ultima visibilia in the nebula of *Orion*, observed at Pulkowa by Dr. Winnecke, simultaneously with me, at Malta. The result of this comparison is expressed in the following terms:—"The preceding comparison of almost simultaneous observations made at Malta and Pulkowa proves evidently that the space-penetrating power of Lassell's new (4-feet) instrument can hardly be estimated superior to that of ours (the Pulkowa 15-inch) refractor; especially if we consider that at Pulkowa the nebula of *Orion* has been observed at a height of only 25°, while at Malta the observations were made at the height of 49°."

I beg leave to add that in 1863, as a great many comparisons made about that time have proved, my eye was still quite of the same strength with that of Dr. Winnecke.

*Pulkowa,* 1876, *November.*

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## *On the Position of the Point of Maximum Brightness on Venus.*

By E. Neison, Esq.

Supposing *Venus* to reflect the solar rays from a dead or unpolished surface, the intensity of illumination of the surface at any point P will be proportional to the cosine of the zenith